Equivalence
The Dream

- Formalizing hallowed notions of position, role and structure
- Society as concrete network of relationships among individuals
  - And social structure is underlying network of positions structuring observed pattern among individuals
- Role freed from essentialist and culturalist definitions and defined in terms of characteristic relations among incumbents of positions, often reciprocally defined
  - Like functional role of species in ecosystem
  - Emergent, relational, roles
Positional Perspective

- Centrality measures one aspect of position
  - Unlike cohesive perspective, we class leaders with leaders, followers with followers, regardless of who they are tied to
- But there are other aspects
  - Not necessarily identified, nor summarizable in non-relational form
Experimental Exchange Nets

- Divvy up 24 points
- Who has what kinds of outcomes?
Node equivalence

- Similarity of position. Structural similarity

- Cohesion. Social (network) proximity
Implicit Hypothesis

- Similar nodes have similar outcomes
  - Occupy same position, then same results
- (Networks with similar structures will also have similar outcomes)
  - Similarly structured teams will have similar performance outcomes
Cohesion vs Equivalence

- Explaining homogeneity – actor similarity
  - Cohesion perspective
    - Diffusion, contagion, influence, imitation, infection
    -_socially proximate actors become similar
  - Equivalence perspective
    - Actors adapt to environments
    - Similar environments result in similar actors

<table>
<thead>
<tr>
<th>cohesion</th>
<th>proximity</th>
<th>melody</th>
<th>longitudinal</th>
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<tbody>
<tr>
<td>equivalence</td>
<td>similarity</td>
<td>harmony</td>
<td>cross – sectional</td>
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</table>

- metonymy
- metaphor
- complementarity
- competition
- knife & fork
- knife & saw
Social proximity & homogeneity

- Why do A and B both know about X?
  - Diffusion from one to the other
  - Diffusion to both from common other
  - Generic principle:
    - Connected nodes become similar
Contagion

- Linked nodes begin to behave or emote in sync
- Chain reactions resulting in collective coherence

Adoption of Smoking

Animation courtesy of http://www.scholarpedia.org/article/Synchronization
Types of contagion processes

- **Locus of agency**
  - If ego acquires alter’s state, was it because of ego’s efforts? Or alter’s? Or both? Or neither?

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<thead>
<tr>
<th>Ego (pull)</th>
<th>Active (push)</th>
<th>Passive</th>
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<tr>
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<td>Accommodation Processes</td>
<td>Mimetic Processes</td>
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<td>E.g., apprenticeship; marriage; congressional politics</td>
<td>E.g., imitation, theft</td>
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<td>Passive</td>
<td>Coercive Processes</td>
<td>Osmotic Processes</td>
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<td>E.g., peer pressure, exercise of power</td>
<td>E.g., language acquisition, schemas, oscillators</td>
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Factors in mimetic processes

- Institutional theory (Dimaggio & Powell, 1983) argues mimesis occurs when there is high need for legitimacy
- Corollary: High status actors more likely to be imitated
- Legitimacy especially important when environment uncertain
Adaptation to environment

- Another generic answer for why A is the same as B is similar response to similar environment
- The state is a response to the opportunities, constraints or necessities imposed by a social environment
  - But the state is not necessarily found in the environment
- Nodes with similar network environments will develop functionally similar features
  - Dolphins and sharks
Social Homogeneity

Adaptation and Contagion

CONTAGION: a node adopts a state found in its environment

An emotional state is transmitted from person to person

ADAPTATION: a node develops a state as a reaction to its environment

What is transmitted is not the state itself – anger is not what flows here
Patterns of homogeneity by mechanism

- **Contagion**: adoption from environment

Homogeneity modeled by proximity and cliques

- **Adaptation**: reaction to environment

Homogeneity modeled by similarity of social environ
A Collection of Concepts

- Structural equivalence
- Automorphic equivalence
- Maximal regular equivalence
- Notes
  - Lattice of regular equivalences
  - Equivalences versus colorations (partitions)
Agenda

- Three equivalence concepts from theoretical point of view
- Computation and implementation
Structural Equivalence
Colorations

- A coloration C is just a partition of nodes.
  - Assignment of nodes to exhaustive, mutually exclusive classes
  - The color of a node v, written C(v) is just the equivalence class it belongs to

- An equivalence is just the relation E induced by a partition

- Is any relation that satisfies 3 conditions:
  - Transitivity: \((a,b), (b,c) \in E \implies (a,c) \in E\)
  - Symmetricity: \((a,b) \in E \iff (b,a) \in E\)
  - Reflexivity: \((a,a) \in E\)
Image Graphs

- Simplified models of a network, usually with a set of rules that describe correspondence between network and model
Neighborhoods

- Neighborhood of $v$, written $N(v)$ is just the set of nodes adjacent to $v$.
- In digraphs, have
  - In-neighborhood $N^i(v)$
    - nodes sending arcs to $v$
  - Out-neighborhood $N^o(v)$
    - Nodes receiving arcs from $v$
Structural Equivalence (simplified)

• $u \equiv v$ if, for any $w$, whenever $u \rightarrow w$ then $v \rightarrow w$, and whenever $w \rightarrow u$ then $w \rightarrow v$

• $C(u) = C(v)$ if $N(u) = N(v)$

• $C(u) = C(v)$ if $N^{\text{out}}(u) = N^{\text{out}}(v)$ and $N^{\text{in}}(u) = N^{\text{in}}(v)$
Structural Equivalence

- Structurally indistinguishable
  - Same degree, centrality, belong to same number of cliques, etc.
  - Only the label on the node can distinguish it from those equivalent to it.
  - Perfectly substitutable: same contacts, resources
- Face the same social environment
  - Similar forces affecting them
- Niche
- Location or position
  - You are your friends
Classical Hypothesis

- Structurally equivalent nodes will have similar internal structures | attitudes | outcomes
- i.e., an explanation for homogeneity
Mechanisms of Homogeneity

- Structural indistinguishability in the context of structural processes
  - Centrality
  - Structural holes
- Similar responses to similar environment
  - adaptation
- Diffusion
  - Through common third parties
Pros and Cons of SE

- **Pros**
  - Captures notions like niche
  - Location or position
    - You are your friends

- **Cons**
  - Confounds similarity with contiguity
  - Not helpful for explaining results of exchange experiments
  - Not a good formalization of social role
    - Mother & father play same role to their kids, but not other parents
    - Can’t use in disconnected graphs
Definition “fails” when structurally equivalent nodes are tied to each other.

\[ C(u) = C(v) \text{ if } N(u) - \{v\} = N(v) - \{u\} \]

*Even better definition is available but is more advanced*
Computation

- Relaxing concepts for real world data
- Two approaches
  - Discrete or blockmodel
    - Partition nodes into mutually exclusive classes such that departures from equivalence model are minimized
  - Profile similarity
    - For each pair of nodes, calculate the degree to which each pair is equivalent
Structural Equivalence

- Profile similarity method
  - Compute similarity/distance between rows of adjacency matrix
    - Correlation
    - Euclidean distance
  - Much argument over handling of diagonals
  - Can then MDS or cluster the resulting proximity matrix
Structural Equivalence

- Blockmodeling approach
  - Optimization method
  - Older Concor method
    - Actually based on profile method
# Blockmodel

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Automorphic Equivalence
Isomorphisms

Mappings:

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<td>e</td>
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A mapping $p$ from one graph to another is an isomorphism if whenever $u$ is tied to $v$, $p(u)$ is tied to $p(v)$.

Isomorphisms are mappings that preserve structure.
Isomorphism

G1

G2

G3

Pg. 33
Automorphisms constitute the “symmetries” of a graph.

An isomorphism from one graph to the same graph is an automorphism.

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<tr>
<th>G</th>
<th>P(G)</th>
<th>P’(G)</th>
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Cycle Notation

- $(1 \ 3) \ (2 \ 4) \ (5)$

- $(a \ b \ d) \ (c)$

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Automorphic Equivalence

- Node $u$ is automorphically equivalent to node $v$ if there exists an automorphism $p$ such that $u = p(v)$
Advantages of AE

- Powerful, fundamental intuitive concept
- Truly structural/positional, not confounded by contiguity
- Captures results of exchange experiments
- Captures essentials of the role concept
- Generalization of structural equivalence that works with disconnected graphs
Problems with Automorphic Equivalence

- A parent with 2 children does not play the same role as one with 3 children
- Extremely difficult to compute
- No obvious way to relax the concept for application to real world data
  - No two nodes are ever AE
Weak Structural Equivalence

- A coloration $C$ of $G(V,E)$ is weakly structural if $C(u)=C(v)$ iff the permutation $p=(u\ v)$ is an automorphism of $G$
(maximal) Regular Equivalence
The Dream (again)

- Formalizing hallowed notions of position, role and structure
- Society as concrete network of relationships among individuals
  - And social structure is underlying network of positions structuring observed pattern among individuals
- Role freed from essentialist and culturalist definitions and defined in terms of characteristic relations among incumbents of positions, often reciprocally defined
  - Like functional role of species in ecosystem
Regular Equivalence

- Two nodes $u$ and $v$ are regularly equivalent if
  - Whenever $u \rightarrow c$, there exists a node $d$ such that $v \rightarrow d$ and $c$ and $d$ are regularly equivalent, and
  - Whenever $c \rightarrow u$, there exists a node $d$ such that $d \rightarrow v$ and $c$ and $d$ are regularly equivalent
- $C(u) = C(v)$ implies $C(N(u)) = C(N(v))$
- Actually, $C(u) = C(v)$ implies $C(N_{out}(u)) = C(N_{out}(v))$ and $C(N_{in}(u)) = C(N_{in}(v))$

Note a coloration is regular iff $C(N(x)) = N(C(x))$
Regular Equivalence
Image graphs

- Image graph of a coloration has colors as vertices and an edge between two colors if there is an edge connecting these colours in the original graph.

Image graph captures the role structure.
Regular Equivalence

- Captures role concept really well
  - Two actors are equivalent if they have the same relations with equivalent others
  - You call American airlines and talk to clerk about booking flight, while I call USAIR and do same with their clerk
    - You and I equivalent because the clerks are equivalent (and they are equivalent because you and I are equivalent)

- Less strict than automorphic
  - Not concerned with quantities of colors
  - Finds more equivalent nodes
Regular Equivalence

- Also captures position in hierarchies well
  - Including trophic group
- Relatively easy to compute (and to relax)
- Easy to generalize to 2-mode data
  - Consumers & brands
    - Segments & positions
    - Identifying category boundaries
- Works well with multiple relations
Hierarchical Position
Multi-dimensional Scaling of the REGE coefficients, with color classes determined by clustering

Image Graph of the food web

Figure 4
Problems with Regular Equivalence

- Often hard to interpret
  - Humans good at understanding pattern similarities, but with social ties we seem to find cohesion easier to see
  - Data sets inappropriate for R.E. analysis
    - Too small, no real roles
- A graph may have multiple colorations that are regular – especially undirected graphs
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21 March 2013
A Family of Regular Equivs

- Every structural equivalence is also regular
- Automorphic is also regular*
- Actually form a lattice
- Somewhat like hierarchical clustering
  - Different levels of resolution

*At least as defined in this presentation. See JMS paper in 1994 for details.
Computation
REGE

- A generalization of the algorithm that uses fuzzy logic
- Still needs directed data with sources or sinks
- Works for multiplex data
- Gives a proximity matrix of degree of equivalence
- In UCINET the previous algorithm is catrege
Ties as the units of structure

- Basic concepts
  - Ties create a unified superordinate structure out of otherwise autonomous agents. Like an organization
    - Super-organic, sui generis
  - The structure has properties distinct from its constituents, like a molecule or bicycle
    - A strategy for scaling up
  - Same constituents organized differently have different properties, like chemical isomers